Geoinformation Issues
Vol. 6, No 1 (6), 5–19/2014

GDQM-PL13 – the new gravimetric quasigeoid model for Poland

Małgorzata Szelachowska
Institute of Geodesy and Cartography, 27 Modzelewskiego St., 02-679, Warsaw, Poland
Tel.: +48 22 3291903, Fax: +48 22 3291950, E-mail: malgorzata.szelachowska@igik.edu.pl

Jan Kryński
Institute of Geodesy and Cartography, 27 Modzelewskiego St., 02-679, Warsaw, Poland
Tel.: +48 22 3291904, Fax: +48 22 3291950, E-mail: jan.krynski@igik.edu.pl

Abstract: The new gravimetric quasigeoid model GDQM-PL13 for Poland was determined. The 1'×1' mean Faye anomalies, deflections of the vertical for the territory of Poland, gravity anomalies from the neighbouring countries and the EGM2008 were used as input data. The remove-compute-restore (RCR) method and the least squares collocation approach with the planar logarithmic covariance function of gravity anomalies were applied. Height anomalies computed from the GDQM-PL13 have been compared with the corresponding ones obtained from GNSS/levelling data at the stations of the POLREF, EUVN and ASG-EUPOS networks and the precise GNSS/levelling control traverse. The new quasigeoid model was also compared with the gravimetric quasigeoid model GDQ08 developed in 2008 for the area of Poland, with the EGM2008, and with the most recent global geopotential model based on GOCE data. The results of the comparison were analysed and the accuracy of the GDQM-PL13 has been assessed and discussed.

Keywords: gravimetric quasigeoid model, least squares collocation, GNSS/levelling data, global geopotential model

Received: 13 March 2015 /Accepted: 9 April 2015

1. Introduction

The figure of the Earth is mathematically described as a geoid, which is defined as an equipotential surface coinciding with the mean ocean surface of the Earth extended through the continents. In the global sense, it is represented by the global geopotential model (GGM) commonly expressed by a set of spherical harmonic coefficients that are determined mainly from satellite missions data. On the regional/local scale, geoid models are usually determined using terrestrial, airborne and marine gravity data as well as altimetry data and topography data with an essential support of GGMs. The precise regional/local geoid/quasigeoid model is desired by the scientific community representing Earth sciences, e.g. geodesy, geophysics and geodynamics. It is also needed by surveyors, especially by the users of the Global Navigation Satellite System (GNSS) for the determination of elevation above sea level.

The direct use of the Stokes or Molodenski integral for the determination of the geoid theoretically involves gravity data from all over the Earth. One of the most popular practical methods of the determination of geoid/quasigeoid models on the regional/local scale is the remove-compute-restore (RCR) method (e.g. Forsberg and Tscherning, 1981; Schwarz et al., 1990; Torge and Müller, 2012), which involves the use of a GGM. The area with the terrestrial gravity data required for developing the geoid model can be limited depending on the spatial resolution of the GGM applied. First, long- and short-wavelength components of the Earth's gravity field determined from the GGM and topography data, respectively, are subtracted from gravity anomalies obtained from terrestrial gravity measurements. In the second step, the residual gravity anomalies are used for the determination of the residual geoid heights. For this stage, there are a few methods which can be applied, e.g. the classical Stokes integral, Fast Fourier Transform technique, etc. It should be noted that the last gravimetric quasigeoid models for Poland, of an accuracy of about 2 cm, were determined on the basis of terrestrial gravity data as well as deflections of the vertical (Krynski and Kloch-Główka, 2009; Łyszkowicz, 2010) using least squares collocation (Moritz, 2000). The last step in the RCR method is restoring a long wavelength component of geoid heights, which is computed from the GGM, and a short wavelength component determined on the basis of topography data.

Faye gravity anomalies, which are frequently used for geoid/quasigeoid modelling, are nothing but free-air gravity anomalies with the effect of topography removed, expressed in terms of terrain corrections. Terrain corrections used for the developing of previous precise gravimetric quasigeoid models for Poland were computed in the framework of the PBZ-KBN-081/T12/2002 project conducted in 2002-2005 by the group of specialists coordinated by the Institute of Geodesy and Cartography (IGiK), Warsaw. The project dedicated to extensive investigations concerning modelling a geoid for Poland with accuracy to a centimetre was supported by the Polish State Committee for Scientific Research. The terrain corrections were determined considering topographic masses around a computational point within the radius of 200 km (Kryński, 2007). Due to the release of the high spatial resolution Earth Geopotential Model 2008 (EGM2008) (Pavlis et al., 2012) which became commonly used for modelling a geoid/quasigeoid on the regional/local scale, new values of terrain corrections, ensuring spectral consistency with the EGM2008, were computed in 2012 for all gravity stations from the gravity database for Poland. These terrain corrections reflect the effect of topography within the radius of 9 km from the computational point, which is consistent with the 5′ spatial resolution of the EGM2008. Their use together with the EGM2008 in the quasigeoid computation process aims to avoid duplication of the effect of topography. The new terrain corrections were calculated using the methodology developed within the project N526 006 32/1079 conducted in 2006-2008 in IGiK, dedicated to precise determination of terrain corrections for the territory of Poland.

Besides new terrain corrections other new components were added to the procedure of developing the new quasigeoid model for Poland. First of all new gravimetric data from the West Tatra Mountains and the Polish part of the Orawa profile obtained in 2012 from the Warsaw University of Technology extended the terrestrial gravity dataset for Poland. The second component concerns the use of atmospheric corrections. It should be noted that the last gravimetric quasigeoid models for Poland (GDQ08, quasi09b, quasi09c) (Krynski and Kloch-Główka, 2009; Łyszkowicz, 2010) were determined without considering atmospheric corrections. In the recent study it has been shown that adding them to gravity anomalies had an essential impact on parameters of covariance functions for gravity anomalies. After applying atmospheric corrections those parameters became much more uniform for the particular areas of Poland and thus one set of parameters of a covariance function for gravity anomalies could be applied for the whole territory of Poland.

The aim of this study is to develop a state-of-the-art gravimetric quasigeoid model for Poland using the available terrestrial gravity data and deflections of the vertical with the support of the EGM2008 as well as to assess the accuracy of that model with the use of all available data.

2. Data used

2.1. Terrestrial gravity data

Terrestrial gravity data used in this study consist of two major datasets of different quality. One contains data from Poland and the second, data from surrounding areas. Gravity anomalies from the area of Poland applied for the determination of the geoid were based on almost 10^6 point gravity data provided by the Polish Geological Institute (Królikowski, 2006) which were unified and reprocessed in IGiK in the framework of the PBZ-KBN-081/T12/2002 project (Kryński, 2007). The accuracy of gravity values for these points was assessed to 0.06 mGal. Free-air gravity anomalies from the neighbouring countries used in this research have been collected, developed and made accessible for geoid modelling by the Department of Planetary Geodesy of the Space Research Centre of the Polish Academy of Sciences (Lyszkowicz, 1994) on a 1.5′×3′ grid. This grid was obtained on the basis of different kinds of gravity data, i.e. the mean 5′×7.5′ free-air gravity anomalies for the areas of Ukraine, the Czech Republic, Slovakia, Hungary and Romania, the mean 5′×5′ and 2′×3′ free-air gravity anomalies for Germany, the mean Bouguer and free-air gravity anomalies and heights on a 8 km × 8 km grid for Belarus, Ukraine and Lithuania and the 8 km × 8 km gravity anomalies for the whole area of interest obtained from Leeds, UK. For preparing a set of 1.5′×3′ mean gravity anomalies for the Baltic Sea, the data from the geophysical marine missions and airborne gravimetry (Kryński, 2007) were used. In addition, gravity data for the West Tatra Mountains (506 points) and the Polish part of the Orawa profile (27 points) provided by the Warsaw University of Technology in 2012 were used in the course of these investigations.

2.2. Deflections of the vertical

The data set of deflections of the vertical for Poland which was used contains 171 astronomical points (Fig. 1). The accuracy of the deflections of the vertical ξ and η was estimated to $\sigma_\xi = 0.2''$ and $\sigma_\eta = 0.3''$, respectively (Bokun, 1961) while in Kamela (1975) the accuracy of both components was evaluated as 0.45″. The accuracy of the deflections of the vertical was also independently assessed by the teams of the Warsaw University of Technology and the University of Warmia and Mazury in the framework of the PBZ-KBN-081/T12/2002 project. In both cases it was estimated to 0.5″ (Kryński, 2007).

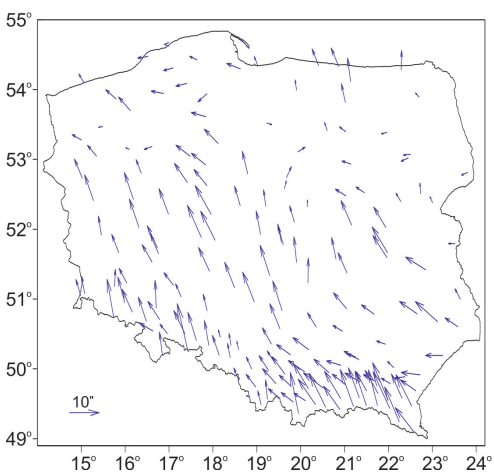

Fig. 1. Distribution of 171 astrogeodetic deflections of the vertical

2.3. Topographic data

The digital elevation model DTED2 (Digital Terrain Elevation Data) for the territory of Poland was developed by the Polish Military according to the NATO-STANAG 3809 standard (NGA, 1996). The resolution of the model is 1″×1″ for the area contained between parallels 49°N and 50°N and 1″×2″ for the area located between parallels 50°N and 55°N. The DTED2 model was developed by digitization of 1:50 000 topographic maps. The vertical accuracy was estimated to 2 m, 4 m, and 7 m, depending on the roughness of topography, with higher values in hilly and mountainous areas.

In addition to the DTED2, the SRTM3 (Shuttle Radar Topography Mission) model has also been used in this study. The SRTM3 is defined as a product of the radar interferometry survey, which covers the areas between parallels 54°S and 60°N. The resolution of this model is 3″×3″. The absolute vertical accuracy of the SRTM3 model was specified as 16 m (Bamler, 1999).

2.4. Global Geopotential Model

The high resolution global geopotential model EGM2008 of 2190 maximum degree and order (d/o) was used as a reference model in the determination of the quasigeoid model in Poland. The performance of the EGM2008 varies from place to place in the world and its accuracy depends on the quality of terrestrial data that have been used when developing this model (Pavlis et al., 2012). The EGM2008 was extensively evaluated over the area of Poland with the use of over 1000 high quality height anomalies on GNSS/levelling sites as well as regional precise quasigeoid models (Krynski and Kloch-Główka, 2009; Lyszkowicz, 2009). Since the high quality mean 5′ × 5′ terrestrial free-air gravity anomalies from Poland were provided for developing the EGM2008, the model shows an excellent performance over the area of Poland; its accuracy was estimated at the level of about 2 cm (Krynski and Kloch-Główka, 2009; Lyszkowicz, 2009).

In this study, one of the recent satellite-only GOCE-based GGMs, i.e. GO_CONS_GCF_2_TIM_

R5 (TIM R5) was also used. The chosen model was developed and released for public use by the European Space Agency (ESA). The TIM R5 GGM is distinguished as a GOCE-only model in a rigorous sense, i.e. no external gravity field information was used, either as a reference model, or for constraining the solution when developing the model (Brockmann et al, 2014). It includes all observations collected during the entire GOCE mission. Further information concerning this GGM can also be found on the ESA's web page https://earth.esa.int and in the International Centre for Global Earth Models (ICGEMs) website http://icgem.gfz-postdam.de.

2.5. GNSS/levelling data

Height anomalies obtained from GNSS/levelling data (Fig. 2) at 184 GNSS/levelling control traverse sites, 315 POLREF (POLish REference Frame), 58 EUVN (European Unified Vertical Network) and 98 ASG-EUPOS (Active Geodetic Network of EUropean POsition Determination System) network sites were used during this study.

The POLREF network was established in 1994-1996. It is a GNSS/levelling network regarded as the extension of the European Terrestrial Reference Frame ETRF. The EUVN was established in 1999. It is the densification of the European Vertical Reference

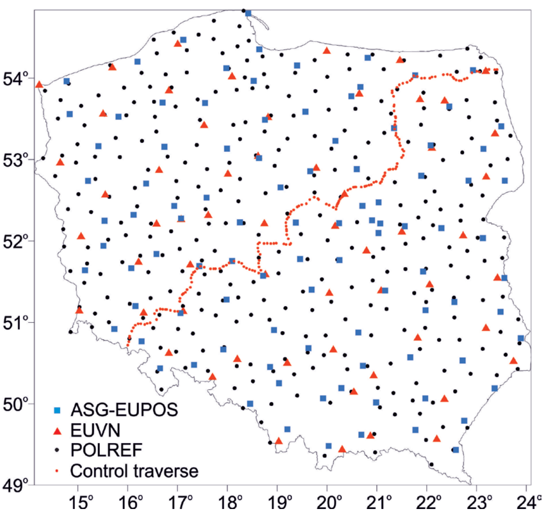

Fig. 2. Distribution of sites of the POLREF, EUVN, ASG-EUPOS networks and the GNSS/levelling control traverse

Network. The 868 km long GNSS/levelling control traverse was established in 2003–2005 by the team of the Institute of Geodesy and Cartography, Warsaw, for the verification and the accuracy estimation of gravimetric quasigeoid models in Poland as well as for the evaluation of interpolation algorithms used for the application of GNSS/levelling quasigeoid models. The ASG-EUPOS was established by the Polish Head Office of Geodesy and Cartography (GUGiK) in 2008 (Bosy et al., 2007, 2008).

The accuracy of height anomalies for GNSS/levelling control traverse sites is estimated to 1–2 cm (Krynski and Lyszkowicz, 2006; Kryński, 2007) while the accuracy of height anomalies for the sites of the POLREF, EUVN and ASG-EUPOS networks is estimated to 3 – 4 cm, 2 cm (e.g. Kryński, 2007), and 1–2 cm, respectively.

3. Methodology

3.1. Determination of mean Faye anomalies

Faye gravity anomaly Δg_F at the computation point P is defined as

$$\Delta g_F = g_P + 0.3086 H_P + \delta g^T - \gamma \qquad (1)$$

where g_P is the "measured" gravity value, H_P is the height of a gravity station, δg^T is the terrain correction computed within the radius $l_{\max}$, which corresponds with the spatial resolution of the global geopotential model (half wavelength) used as a reference geopotential model in the gravimetric quasigeoid model computation process. The terrain correction can be presented in a planar approximation as follows

$$\delta g^T \approx G\rho \int_S \int_{H_P}^{H} \frac{h}{\left(l^2 + h^2\right)^{2/3}} dh dS \approx \frac{G\rho}{2} \int_S \frac{\left(H - H_P\right)^2}{l^3} dS \qquad (2)$$

where S represents the Earth's surface projected to a plane, l is a planar distance between the point P and the centre point of element dS, H is the height of dS element, G is a gravitational constant, ρ = 2.67 g/cm^3 is the density of terrain (of a lithosphere).

The value of γ is computed from the Somigliana closed-form formula or the normal gravity γ on the reference ellipsoid at latitude φ

$$\gamma = \frac{\gamma_e\left(1 + k\sin^2\varphi\right)}{\left(1 - e^2\sin^2\varphi\right)^{1/2}} \tag{3}$$

with γ_e, k and e – parameters of GRS80 ellipsoid, i.e. γ_e = 978032.67715 mGal (normal gravity at the equator of the ellipsoid); k = 0.001931851353 (a derived constant); and e^2 = 0.0066943800229 (e – the first eccentricity).

Mean Faye gravity anomalies were determined using Bouguer anomalies as substantially smoother for the interpolation. The Bouguer anomaly Δg_B at the computation point P is defined as follows

$$\begin{aligned} \Delta g_B &= g_P + 0.3086H_P + \delta g^T - \gamma - 0.0419\rho H_P = \\ &= \Delta g_F - 0.0419\rho H_P \end{aligned} \tag{4}$$

where the expression $0.0419\rho H_P$ represents the impact of a plate of thickness H_P and density ρ (called a Bouguer plate) on a gravity value.

Atmospheric corrections applied to gravity anomalies before the quasigeoid modelling process can be calculated with the use of the following formula (Wichiencharoen, 1982)

$$\begin{aligned} \Delta g^A = (0.8658 - 9.727 \times 10^{-5}H_P + \\ + 3.482 \times 10^{-9}H_P^2)\ \text{mGal} \end{aligned} \tag{5}$$

3.2. Determination of a gravimetric quasigeoid model using the RCR method

The concept of the RCR method is first removing from the data, i.e. gravity anomalies and deflections of the vertical, the effects coming from the global geopotential model, then computing the residual geoid heights from the residual gravity field functionals, and finally restoring the effect of the global geopotential model in geoid heights. This procedure allows only the data from the area within the radius corresponding to the spatial resolution of the global geopotential model used in a geoid model determination process to be taken into account. The scheme of the determination of height anomaly ζ using gravity anomalies Δg for the area of interest and the surrounding areas, deflections of the vertical ξ, η for the study area and a GGM as input data in the RCR technique with the least squares collocation method for the determination of medium wavelength component N_{res} is shown in Figure 3.

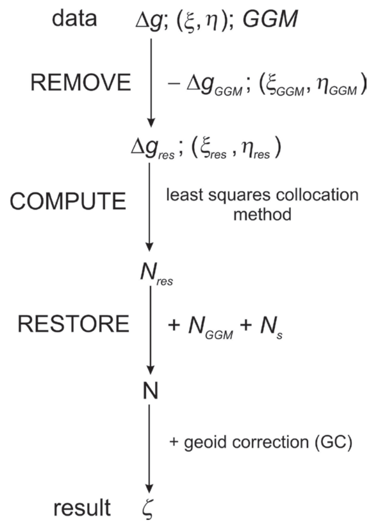

Fig. 3. Height anomaly determination scheme using the RCR method

3.2.1. Remove step of the RCR method – removing long-wavelength components from gravity anomalies and deflections of the vertical with the use of a GGM

The residual gravity anomalies Δg_{res} are obtained from gravity anomalies Δg, particularly Faye anomalies Δg_F (Section 3.1), after removing the long-wavelength component Δg_{GGM} of the Earth's gravity field calculated from a GGM

$$\Delta g_{res} = \Delta g - \Delta g_{GGM} - \Delta g_s \tag{6}$$

where

$$\begin{aligned} \Delta g(r,\varphi,\lambda) &= \frac{GM}{r^2\gamma\cos\varphi}\sum_{n=2}^{N\max}\left(\frac{a}{r}\right)^n \times \\ &\times \sum_{m=0}^{n}(\Delta\bar{C}_{nm}\cos m\lambda + \Delta\bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\varphi) \end{aligned} \tag{7}$$

with r, φ, λ – geocentric coordinates of a point P; GM – the product of the Newtonian gravitational constant G and the mass of the Earth M; a – the semi-major axis of the reference ellipsoid; $N_{\max}$ – the applied maximum degree of a global geopotential model; $\Delta\bar{C}_{nm}$, $\Delta\bar{S}_{nm}$ – residual harmonic coefficients, being defined as differences between the coefficients of the actual and normal gravity field of degree n and order m; and $\bar{P}_{nm}(\sin\varphi)$ – fully normalized associated Legendre functions of degree n and order m (Torge and Müller, 2012). In the case of using

Faye anomalies the effect of the short-wavelength signal Δg_S is removed by applying terrain corrections.

Residual deflections of the vertical ξ_{res}, η_{res} are computed from the deflections of the vertical ξ, η (Section 2.2) after removing the long-wavelength component ξ_{GGM}, η_{GGM} of the Earth's gravity field calculated from a GGM

$$\xi_{res} = \xi - \xi_{GGM} \tag{8}$$

$$\eta_{res} = \eta - \eta_{GGM} \tag{9}$$

where ξ_{GGM}, η_{GGM} are expressed as follows (Torge and Müller, 2012, p. 273)

$$\xi_{GGM}(r,\varphi,\lambda) = \frac{GM}{r^2\gamma}\sum_{n=2}^{N_{\max}}\left(\frac{a}{r}\right)^n \times \\ \times\sum_{m=0}^{n}(\Delta\overline{C}_{nm}\cos m\lambda + \Delta\overline{S}_{nm}\sin m\lambda)\frac{d\overline{P}_{nm}(\sin\varphi)}{d\varphi} \tag{10}$$

$$\eta_{GGM}(r,\varphi,\lambda) = \frac{GM}{r^2\gamma\cos\varphi}\sum_{n=2}^{N_{\max}}\left(\frac{a}{r}\right)^n \times \\ \times\sum_{m=0}^{n}(-m\Delta\overline{C}_{nm}\sin m\lambda + m\Delta\overline{S}_{nm}\cos m\lambda)\overline{P}_{nm}(\sin\varphi) \tag{11}$$

with γ – the normal gravity.

3.2.2. Compute step of the RCR method – determining residual geoid heights

Residual geoid heights N_{res} can be computed from residual gravity anomalies Δg_{res} and residual deflections of the vertical ξ_{res}, η_{res} using the least squares collocation (LSC) method (Moritz, 1980)

$$N_{res} = \mathbf{C}_{N_{res}l}\mathbf{C}_l^{-1}\mathbf{l} \tag{12}$$

where $\mathbf{l}$ is the vector of observations, $\mathbf{C}_{ll}$ is the auto-covariance matrix of observations, $\mathbf{C}_{Nresl}$ is the matrix of the cross-covariance between residual geoid heights N_{res} and observations. When the geoid model is computed on the basis of gravity anomalies and deflections of the vertical, the vector of observations $\mathbf{l}$ can be written as

$$\mathbf{l} = \begin{bmatrix}\mathbf{l}_{\xi_{res}\eta_{res}} \\ \mathbf{l}_{\Delta g_{res}}\end{bmatrix} \tag{13}$$

where the vectors $\mathbf{l}_{\zeta_{res}\eta_{res}}$ and $\mathbf{l}_{\Delta g_{res}}$ contain residual deflections of the vertical and residual gravity anomalies, respectively.

The matrices $\mathbf{C}_{ll}$ and $\mathbf{C}_{Nresl}$ can be obtained by applying the planar logarithmic covariance function model, which is described by three parameters: C_0 – the variance of the gravity anomalies, which plays the role of a scale coefficient; D – describes the threshold of the high frequency part of the gravimetric signal; and T – defines the threshold of the low frequency part of the gravity signal (Forsberg, 1987). These parameters are obtained by fitting the covariance functions obtained from the planar logarithmic covariance function model (see Forsberg, 1987) to the empirical covariance functions obtained from residual gravity anomalies for the whole territory of Poland.

3.2.3. Restore step of the RCR method – restoring the effect of the GGM and topography

Gravimetric geoid heights computed using the remove-compute-restore method are given as

$$N = N_{GGM} + N_{res} + N_{ind} \tag{14}$$

where N_{GGM} is the reference geoid height determined from the GGM (Torge and Müller, 2012)

$$N_{GGM}(R,\varphi,\lambda) = N_0 + \frac{GM}{R\gamma_0}\sum_{n=2}^{N_{\max}}\left(\frac{a}{R}\right)^n \times \\ \times\sum_{m=0}^{n}\Delta\overline{C}_{nm}\cos m\lambda + \Delta\overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\varphi) \tag{15}$$

with R being the mean radius of the Earth and γ_0 the normal gravity on the ellipsoid.

The parameter N_0 in Eq.(15) reflecting the effect of the difference in the mass of the Earth used in the International Earth Rotation and Reference Systems Service IERS Convention and the Geodetic Reference System 1980 GRS80 ellipsoid is expressed as follows (Heiskanen and Moritz, 1967)

$$N_0 = \frac{GM - GM_0}{R\gamma_m} - \frac{W_0 - U_0}{\gamma_m} \tag{16}$$

where M_0 is the mass of the reference ellipsoid, U_0 is the gravity potential of the ellipsoid, R is the mean radius of the reference ellipsoid and γ_m is the mean normal gravity on the reference ellipsoid

(979 800 mGal). The values of these parameters are related to the GRS80 (Moritz, 2000). On the other hand, W_0 is the gravity potential of the Earth, which together with M is defined among the numerical standards of the IERS Conventions (McCarthy and Petit, 2004).

Since the displacement of the topographic masses applied in gravity reductions changes the gravitational potential (the so called indirect effect), the computed surface $N_{GGM} + N_{res}$ is not exactly the geoid but a slightly different surface called the co-geoid. The vertical distance between the geoid and co-geoid in the case of using Faye anomalies can be determined using the formula (Omang and Forsberg, 2000)

$$N_{ind} = -\frac{\pi G\rho}{\gamma_m} H_P^2 - \frac{G\rho}{6\gamma_m} \iint_S \frac{H^3 - H_P^3}{l^3} dS \quad (17)$$

For flat areas the component N_{ind} in the case of using Faye anomalies can practically be computed with sufficient accuracy using the following simplified formula (Grushinsky, 1976).

$$N_{ind} = -\frac{\pi G\rho}{\gamma_m} H_P^2 \quad (18)$$

3.4. Geoid to quasigeoid separation

The geoid height at the point P is converted to the height anomaly at P as follows (Torge and Müller, 2012)

$$\zeta_P = N_P - \frac{\Delta g_B}{\gamma_m} H_P \quad (19)$$

4. Results

4.1. Determination of 1'×1' mean Faye anomalies for the area of Poland

The gravity database for Poland consisting of almost one million gravity stations was the main source of data for the computation of Faye anomalies. Heights of gravity stations and the respective gravity values are given in Figure 4 and Figure 5, respectively. New terrain corrections for all gravity stations from the gravity database for Poland (Fig. 6) were calculated by the team of the Institute of Geodesy and Cartography in 2012. The effect of topography was computed for each gravity station using topography data from DTED2 and SRTM3 digital elevation models within the radius of 9 km, which corresponds to the spatial resolution of the EGM2008. New gravity data from the West Tatra Mountains and the Polish part of the Orawa profile provided by the Warsaw University of Technology partly filled the gap in gravity data in the area of the Tatra Mountains. In this study, a grid of 1′×1′ Faye gravity anomalies for the territory of Poland has been generated.

Faye anomalies are not smooth at all, they depend on topography and they should not be interpolated (Forsberg, 2005). A grid of mean Faye anomalies can, however, be computed through Bouguer anomalies which are smoother and thus more suitable for interpolation. In such a case point Bouguer anomalies for gravity stations described in section 2.1 are gridded using the kriging interpolation method. Then mean 1′×1′ Bouguer anoma-

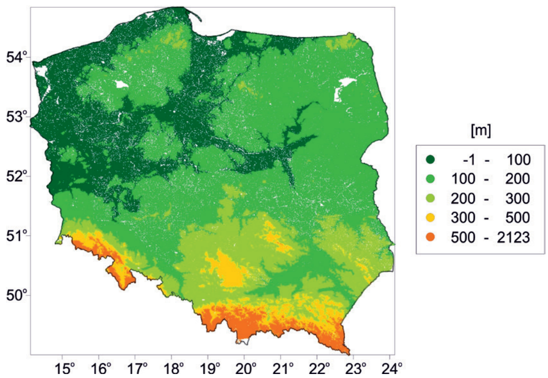

Fig. 4. Heights of gravity stations

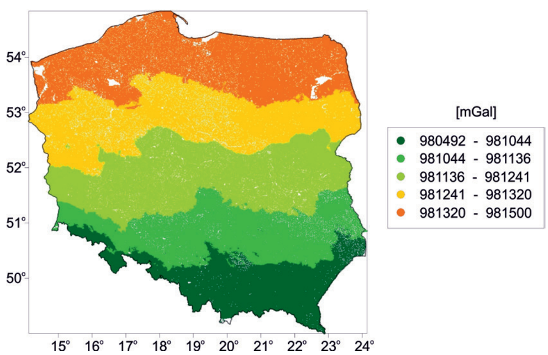

Fig. 5. Gravity values at gravity stations

Geoinformation Issues
Vol. 6, No 1 (6), 5–19/2014

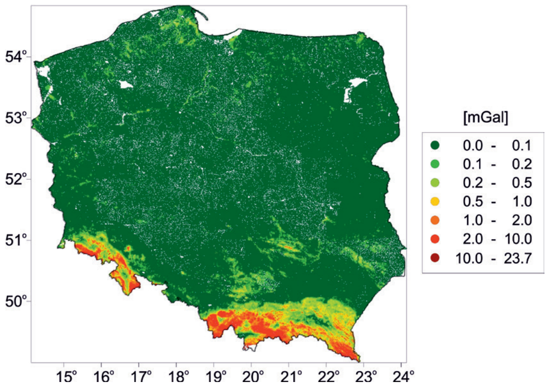

Fig. 6. Terrain corrections

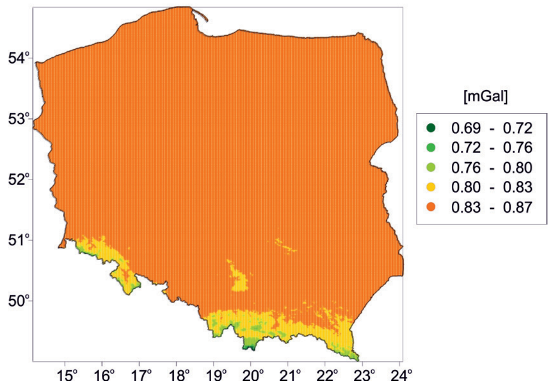

Fig. 7. Atmospheric corrections in 1'×1' grid

lies are calculated. In the next step the effect of the Bouguer plate on gravity is restored and finally mean 1′×1′ Faye anomalies are obtained. In the case of the investigations presented the average heights obtained from the DTED2 model were used for restoring the Bouguer plate. In addition, atmospheric corrections (Fig. 7) were added to Faye anomalies. Final mean 1′×1′ Faye gravity anomalies with atmospheric corrections for the territory of Poland as well as gravity anomalies described in Section 2.1 for the area surrounded are shown in Figure 8.

4.2. Determination of the gravimetric quasigeoid model

Residual gravity anomalies Δg_{res} (Fig. 9) were computed for the whole area of interest, i.e. 47°– 57°N and 11° – 27°E, from gravity anomalies Δg (for the territory of Poland the subset of Δg consists of mean 1′×1′ Faye gravity anomalies Δg_F described in Section 4.1) with the use of Eq. (6), after removing the component Δg_{GGM} of gravity anomalies determined from the EGM2008 using Eq. (7). Statistics of Δg_{res}, Δg_F, Δg_{GGM} for the territory of Po-

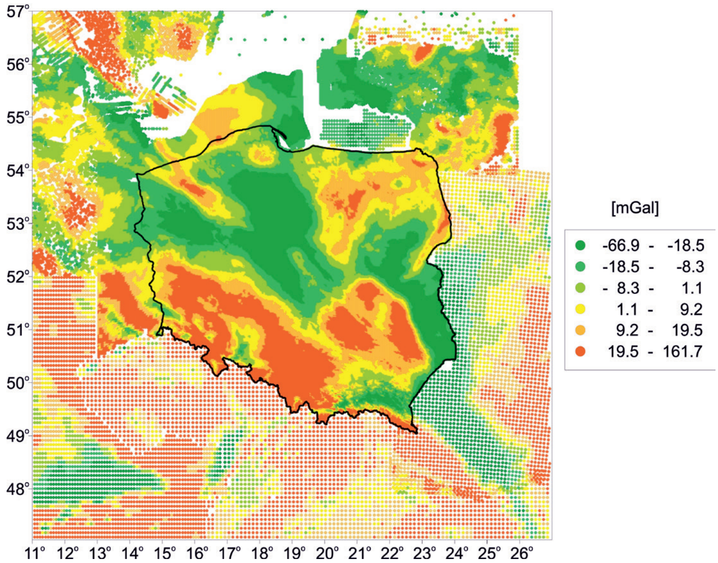

Fig. 8. Gravity anomalies, 1′×1′ mean Faye anomalies for the area of Poland

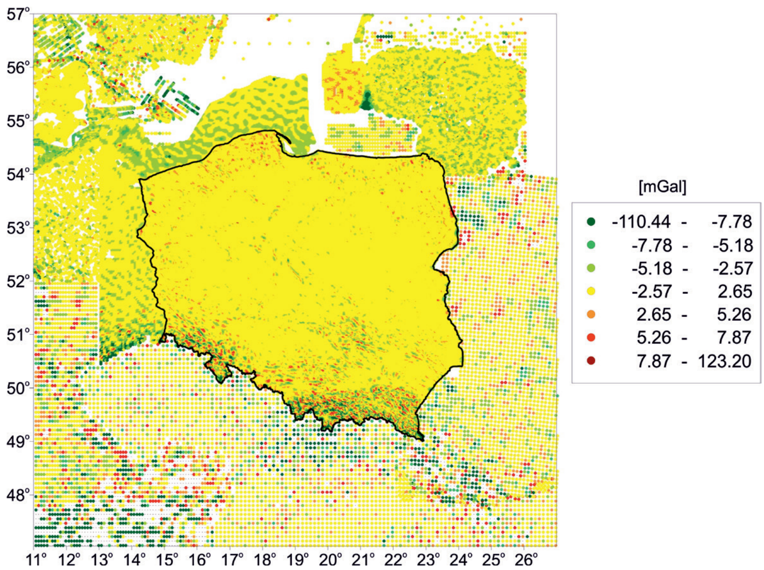

Fig. 9. Residual gravity anomalies

land are given in Table 1. The standard deviation of residual gravity anomalies is 10 times smaller than the one of mean Faye anomalies. It is caused by the fact that the EGM2008 reproduces extremely well the gravity field over the area of Poland (Krynski and Kloch-Główka, 2009).

A set of 1′×1′ residual gravity anomalies for the territory of Poland was used for computing the empirical covariance function. Parameters of a planar logarithmic covariance function of gravity anomalies were obtained using the *gpfit* program from the GRAVSOFT package (Tscherning et al., 1992). The estimated values of these parameters are as follows: $\sqrt{C_0}$ = 2.61 mGal, D = 2 km, T = 2 km. Empirical and fitted covariance functions for gravity anomalies are shown in Figure 10. The correlation length is equal to 3 km.

The planar logarithmic covariance function with the estimated parameters $\sqrt{C_0}$, D and T, was applied to determine the residual geoid heights N_{res} on a 1.5′×3.0′ grid using the residual gravity anomalies Δg_{res} for the whole area investigated, together with residual deflections of the vertical ξ_{res}, η_{res}. Calculations were performed with the least squares collocation method using the *gpcol* program from the GRAVSOFT package (Tscherning et al., 1992). The EGM2008 model up to d/o 2190 was used as a reference GGM using Eq. (15). Sta-

Table 1. Statistics of mean 1′×1′ Faye anomalies, gravity anomalies from EGM2008 and residual gravity anomalies for the territory of Poland [mGal]

Gravity anomalies	Min	Max	Mean	Std. dev.
Δg_{GGM}	–56.63	155.70	0.95	20.66
Δg_{F}	–57.63	158.98	0.99	20.82
Δg_{res}	–65.08	40.69	0.04	2.61

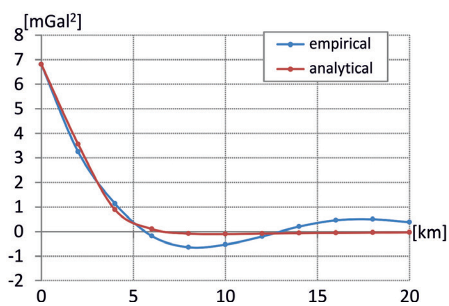

Fig. 10. Empirical gravity anomalies covariance functions (blue) and fitted ones (red) with parameters $\sqrt{C_0}$ = 2.61 mGal, D = 2 km, T = 2 km

tistics of residual, reference and original values of input data as well as N_{res} are shown in Table 2.

The standard deviations of gravity anomalies and deflections of the vertical drop significantly after subtracting reference values calculated from the EGM2008. The standard deviations of residual components are on average five times smaller than those for corresponding reference values. It reflects that the EGM2008 represents the gravity field well for the territory of Poland.

In the next step the indirect effect N_{ind} was computed using Eq. (17) for mountainous areas and Eq. (18) for the rest of the country. Finally height anomalies were obtained transforming the geoid to the quasigeoid with the Eq. (19). Maps with the results of consecutive steps of the quasigeoid computational process are presented in Figure 11. The determined quasigeoid model named GDQM-PL13 (Fig. 11f) has been stored in the form of a grid of 1.5′×3′ resolution for the area between the parallels 47° and 57° and meridians 11° and 27°. The greatest contribution to the GDQM-PL13 comes from the reference model EGM2008 which represents the long wavelength component of the gravity field (Fig. 11a).

Table 2. Statistics of gravity anomalies and deflections of the vertical as well as computed residual geoid heights for the whole area investigated

Gravity functional component	Min	Max	Mean	Std. dev.
Δg [mGal]	–66.90	161.64	0.87	20.05
Δg_{GGM} [mGal]	–75.91	219.54	1.21	19.95
Δg_{res} [mGal]	–110.44	123.15	–0.34	3.23
ξ [arcsec]	–2.70	15.15	4.91	3.39
ξ_{GGM} [arcsec]	–16.82	2.65	–5.10	3.39
ξ_{res} [arcsec]	–2.49	2.21	–0.15	0.69
η [arcsec]	–3.68	11.41	4.64	2.69
η_{GGM} [arcsec]	–12.80	4.89	–4.31	2.74
η_{res} [arcsec]	–2.34	3.81	0.13	0.72
N_{res} [m]	–0.228	0.241	–0.011	0.026

The residual geoid heights N_{res} shown in Figure 11b represent the medium/short wavelength gravity

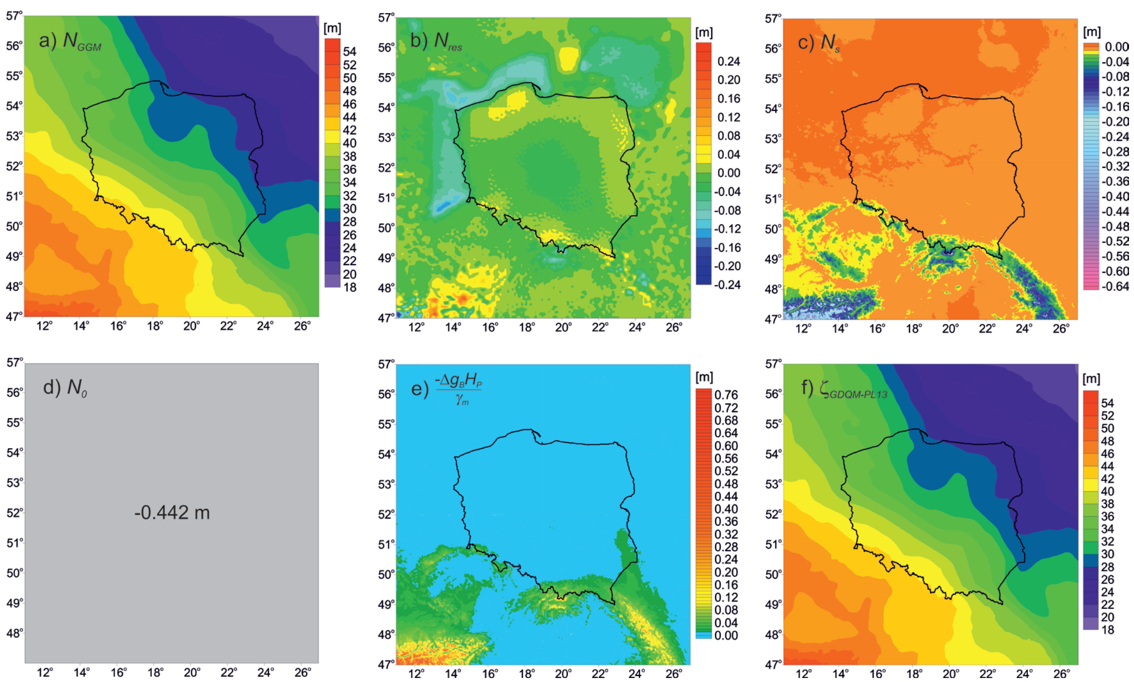

Fig. 11. Components of GDQM-PL13: (a) N_{GGM} (Eq. 14), (b) N_{res} (Eq. 14), (c) N_s (Eq. 14), (d) N_0 (Eq. 16), (e) $\zeta_P - N_P$ (Eq. 19), (f) $\zeta_{GDQM-PL13}$ (Eq. 19)

signal from the terrestrial gravity data, starting from 2190 d/o. They range from –24 cm to +26 cm for the whole area of interest and only from –17 cm to +8 cm for the area of Poland. Such small values result from using high quality 5′×5′ free-air anomalies from the area of Poland when developing the EGM2008. The indirect effect N_{ind} (Fig. 11c) and geoid to quasigeoid correction (Fig. 11e) for the whole area of interest range from –65 cm to 20 cm, and from 0 to 77 cm, respectively. For the majority of the area of Poland these values do not exceed 1 cm. They reach 17 cm and 19 cm, respectively, for an indirect effect and geoid to quasigeoid correction only in high mountains.

4.2. Accuracy assessment of the GDQM-PL13

Height anomalies from the GDQM-PL13 were compared with the corresponding ones from the POLREF, EUVN, ASG-EUPOS networks and GNSS/levelling control traverse (Table 3 and Figure 12).

$$\Delta\zeta = \zeta_{GNSS/levelling} - \zeta_{GDQM-PL13} \tag{20}$$

Standard deviations of differences $\Delta\zeta$ for GNSS/levelling control traverse, EUVN as well as ASG-EUPOS, and POLREF sites are 1.4-1.6 cm, 1.8 cm and 2.2 cm, respectively. Mean values are in the range from 7.4 to 10.5 cm.

Table. 3. Statistics of differences $\Delta\zeta$ between height anomalies from GNSS/levelling measurements and from the GDQM-PL13 model [m]

GNSS/level. sites	Number of Pt.	Min	Max	Mean	Std. dev.
Contr. trav. 1st order	44	0.064	0.124	0.097	**0.014**
Contr. trav. 2nd order	140	0.040	0.127	0.083	**0.016**
EUVN	58	0.060	0.145	0.097	**0.018**
ASG-EUPOS	98	0.041	0.133	0.074	**0.018**
POLREF	315	0.032	0.169	0.105	**0.022**

The assessed accuracy, in terms of the standard deviation of differences of height anomalies obtained from the GDQM-PL13 model, exceeds 2 cm only for the POLREF sites. For the rest of the GNSS/levelling data, i.e. EUVN and ASG-EUPOS sites as well as GNSS/levelling control traverse, the assessed accuracy is below 2 cm with the minimum standard deviation of differences of 1.4 cm for the 1st order GNSS/levelling control traverse sites. The distribution of differences of height anomalies presented in Figure 12 is similar for all sets of data. The standard deviations shown in Table 3 seem to reflect the quality of groups of GNSS/

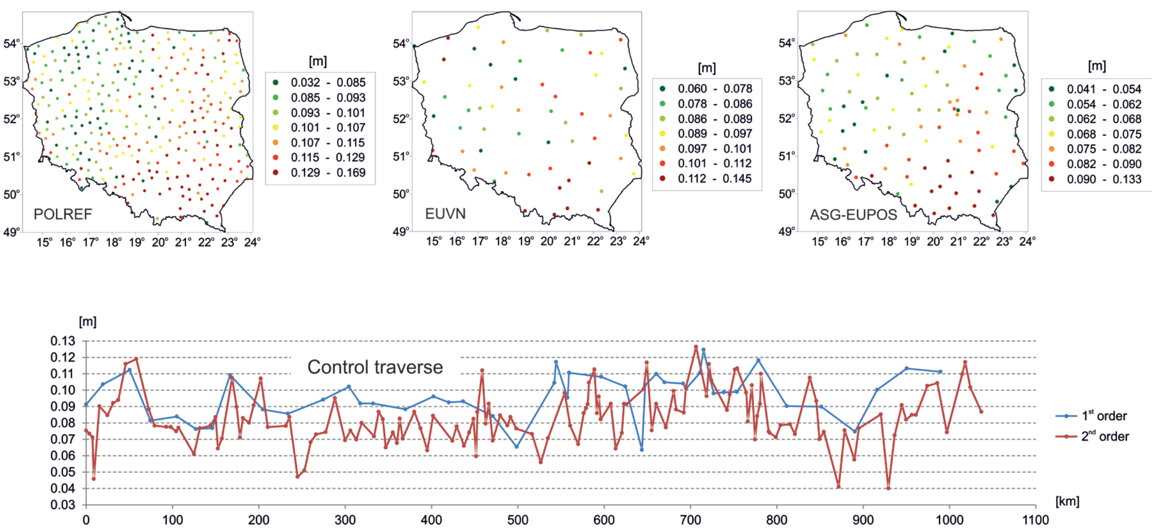

Fig. 12. Comparison of height anomalies determined from the GDQM-PL13 with height anomalies from the POLREF, EUVN, ASG-EUPOS GNSS/levelling networks and GNSS/levelling control traverse sites

Geoinformation Issues
Vol. 6, No 1 (6), 5–19/2014

levelling sites. They may indicate that the POLREF sites are not suitable to validate precise gravimetric quasigeoid models. The above example proves that precise geoid/quasigeoid models should rather be used for checking the accuracy of GNSS/levelling data. An alternative source of data is needed for validation of geoid/quasigeoid models.

The new quasigeoid model GDQM-PL13 was also compared with other quasigeoid models. Differences between height anomalies from the GDQM-PL13 model and the GDQ08 model – one of the latest gravimetric quasigeoid models for Poland computed using the least squares collocation approach with gravity anomalies and deflections of the vertical as input data – were calculated. Moreover, the GDQM-PL13 was also compared with global geopotential models: EGM2008 and TIM RL05 based on GOCE data truncated at maximum degree/order (d/o) 200, taking into consideration their spatial resolution. Results of comparisons are given in Table 4 and in Figure 13.

No bias between the GDQM-PL13 and the EGM2008 is observed. The bias of 1 cm between the GDQM-PL13 and the GDQ08 may indicate the use of the atmospheric correction in modelling the GDQM-PL13. In the case of the TIM RL05 the bias of 3 cm reflects mainly the effect of the omitted gravity signal, i.e. from d/o 201 onward, in the GGM. It may also indicate the difference between the EGM2008 and TIM R5 in the long wavelength component (i.e. up to d/o 200).

Table 4. Statistics of differences between height anomalies from the GDQM-PL13 and the corresponding ones from the GDQ08, EGM2008 and TIM RL05 [m]

Model	Number of Pt.	Min	Max	Mean	Std. dev.
GDQ08	32836	–0.17	0.09	–0.01	0.011
EGM2008	5917	–0.17	0.07	0.00	0.016
TIM RL05	52	–0.68	0.87	–0.03	0.294

The standard deviation of differences between height anomalies computed from the GDQM-PL13 and the corresponding ones computed from the GDQ08 is 1.1 cm. It can be interpreted as the effect of using new values of terrain corrections compatible with the resolution of the EGM2008 model and applying atmospheric corrections during the quasigeoid computation process as well as adding new gravity data in mountainous areas when modelling the GDQM-PL13. Statistics of differences between height anomalies obtained from the GDQ08 and GNSS/levelling data can be found in Krynski and Kloch-Główka (2009). The standard deviations of those differences are of 3.3 cm, 2.1 cm, and 1.7 cm for the POLREF, EUVN networks sites and the 1st order GNSS/levelling control traverse sites, respectively. For the POLREF network a 1 cm improvement is observed. For the EUVN network and the GNSS/levelling control traverse sites only a 3 mm reduction in the standard deviation of height anomaly is observed.

The standard deviation of differences between height anomalies determined from the GDQM-PL13 and the respective ones from the EGM2008 equals 1.6 cm. It reflects the effect of the gravity signal contained within the wavelength range from 5′ to 1′.

The standard deviation of differences between height anomalies computed from the GDQM-PL13 and GOCE-based GGM TIM RL05 truncated at d/o 200 on a grid of resolution of the GOCE mission (100 km half-wavelength) equals to 30 cm. Its large value results from the fact that the TIM RL05 GGM includes only the long wavelength component (up to d/o 200) of the Earth's gravity field.

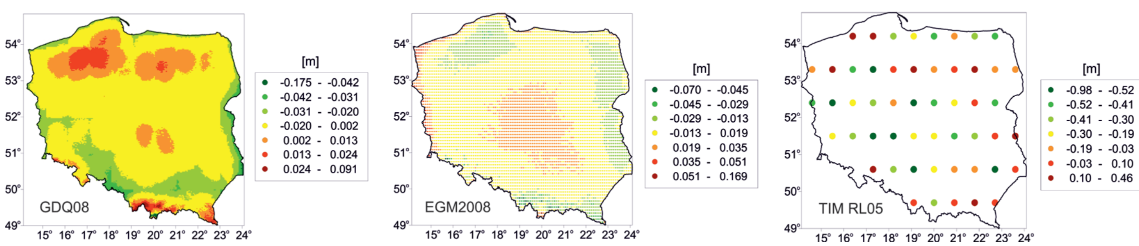

Fig. 13. Comparison of the GDQM-PL13 model with other quasigeoid models

5. Summary and conclusions

Over the last years the Institute of Geodesy and Cartography, Warsaw, has acquired new data and developed a strategy for improving the gravimetric quasigeoid model for the area of Poland. The new gravimetric quasigeoid model GDQM-PL13 for Poland has thus been computed.

First, mean 1′×1′ Faye anomalies for the area of Poland, being the most important input data in the process of determination of the GDQM-PL13 quasigeoid model, were computed. The new quasigeoid model for Poland was determined also on the basis of deflections of the vertical for the Poland area strengthened with free-air gravity anomalies for neighbouring countries. The RCR method with the use of the EGM2008 as a reference GGM and the least squares collocation approach was implemented to compute the GDQM-PL13.

The EGM2008 reproduces the gravity field in the area of Poland with an accuracy of 2–3 cm in height anomaly (Krynski and Kloch-Główka, 2009) so the planar logarithmic covariance function of gravity anomalies is accurate enough to be applied in the compute step of the RCR method. The estimated parameters of the planar logarithmic model of covariance function based on residual 1′×1′ mean Faye anomalies from the area of Poland are as follows: $\sqrt{C_0}$ = 2.61 mGal, D = 2 km, T = 2 km with the correlation length equal to 3 km. Their magnitude is the consequence of small residual gravity anomalies. Also the accurate GGM used causes that the residual geoid heights N_{res} are small for Poland; they range from –17 to +8 cm with zero mean value and the standard deviation of 1.5 cm.

An accuracy assessment of the new gravimetric quasigeoid model GDQM-PL13 with the use of precise GNSS/levelling data proved its high quality. The fit of the geoid heights from the GDQM-PL13 to the corresponding ones of 1st and 2nd order sites of GNSS/levelling control traverse, the EUVN, ASG-EUPOS, and POLREF networks sites represented by the standard deviations of respective differences is of 1.4 cm, 1.6 cm, 1.8 cm, 1.8 cm, and 2.2 cm, respectively.

The new gravimetric quasigeoid model GDQM-PL13 was also evaluated by comparing it with the GDQ08 quasigeoid model as well as the GGMs EGM2008 and TIM RL05, on the grids corresponding to the spatial resolution of the models. The fit of geoid heights from the GDQM-PL13 to the corresponding ones from the GDQ08, EGM2008, and TIM RL05 in terms of the standard deviations is 1.1 cm, 1.6 cm, and 30 cm, respectively. No systematic error of the GDQM-PL13 with respect to the EGM2008 is observed. For the GDQ08 and TIM RL05 the biases of 1 cm and 21 cm were detected.

The greatest differences between height anomalies are obtained in mountainous areas both when comparing the GDQM-PL13 with GNSS/levelling data and with other quasigeoid models. There is thus a need for more detailed research on the determination of a precise quasigeoid model in hilly and mountainous areas.

Conscientious verification of gravity data from neighbouring countries used for the computation of the new quasigeoid model is required, especially for the Baltic Sea. The use of more accurate gravity data from the Czech Republic and Slovakia is recommended to increase the accuracy of the quasigeoid model in hilly and mountainous areas near the southern border of Poland.

The results obtained indicate a growing problem with the validation of the precise quasigeoid model. The accuracy of the computed GDQM-PL13 model, which is below 2 cm seems better than the accuracy of 2-3 cm of GNSS/levelling data. The search for an alternative source of data for the validation of quasigeoid models is recommended. The investigations connected with the use of absolute gravity measurements data for such validation should be the subject of further research.

Acknowledgments

The research was conducted within the framework of the statutory project “Problems of geodesy and geodynamics” of the Institute of Geodesy and Cartography, Warsaw, financially supported by the Polish Ministry of Science and Higher Education. The authors would also like to acknowledge the team from Warsaw University of Technology for providing gravity data for the West Tatra Mountains and the Polish part of the Orawa profile.

References

Bamler R., (1999): *The SRTM Mission: A World-Wide 30 m Resolution DEM form SAR Interfero-*

metry in 11 Days, In D. Fritsch, D. Spiller (eds.), Photogrammetric Week 99, Wichmann Verlag, Heidelberg, pp. 145–154.

Bokun, J., (1961): *Geoid determination in Poland on the base of astrogravimetric and gravity data* (in Polish), Proceedings of the Institute of Geodesy and Cartography, t. VIII, z. 1(17).

Bosy J., Graszka W., Leonczyk M., (2007): *ASG-EUPOS – a Multifunctional Precise Satellite Positioning System in Poland*, International Journal on Marine Navigation and Safety of Sea Transportation, Vol. 7, No 4, pp. 371–374.

Bosy J., Oruba A., Graszka W., Leonczyk M., Ryczywolski M., (2008): *ASG-EUPOS densification of EUREF Permanent Network on the territory of Poland*, Reports on Geodesy, No 2(85), pp. 105–112.

Brockmann J.M., Zehentner N., Höck E., Pail R., Loth I., Mayer-Gürr T., Schuh W.D., (2014): *EGM TIM RL05: An Independent Geoid with Centimeter Accuracy Purely Based on the GOCE Mission*; Geophysical Research Letters, Wiley, doi: 10.1002/2014GL061904.

Forsberg R., Tscherning C.C., (1981): *The Use of Heights Data in Gravity Field Approximation*, Journal of Geophysical Research (86), pp. 7843–7854.

Forsberg R., (1987): *A new covariance model for inertial gravity and gradiometry*, Journal of Geophysical Research, Vol. 92, No B2, pp. 1305–1310.

Forsberg R., (2005): *Terrain effects in geoid computation*, International School for the determination and use of the geoid, Budapest, 31 January – 5 February, 2005.

Grushinsky N.P., (1976): *Theory of the earth figure*, Nauka, Moscow.

Heiskanen W.A., Moritz H., (1967): *Physical geodesy*, W.H. Freeman and Company, San Francisco.

Kamela Cz., (1975): *Report of experts on the improvement of horizontal control network in Poland* (in Polish), Główny Urząd Geodezji i Kartografii, Warsaw.

Krynski J., Lyszkowicz A., (2006): *Centimetre quasigeoid modelling in Poland using heterogeneous data*, Proceedings of the 1st International Symposium of the International Gravity Field Service (IGFS), 28 August – 1 September, 2006, Istanbul, Turkey, pp. 123–127.

Kryński J., (2007): *Precise quasigeoid modelling in Poland–Results and accuracy estimation* (in Polish), Monographic series of the Institute of Geodesy and Cartography, Nr 13, Warsaw, Poland, (266 pp).

Krynski J., Kloch-Główka G., (2009): *Evaluation of the performance of the new EGM2008 global geopotential model over Poland*, Geoinformation Issues, Vol. 1, No 1, Warsaw, pp. 7–17.

Królikowski C., (2006): *Gravimetric survey of Poland – its value and importance for Earth sciences* (in Polish), Biuletyn Państwowego Instytutu Geologicznego, 420, Warszawa (104 pp.).

Lyszkowicz A., (1994): *Description of the algorithm for the determination of geoid in Poland, gravity data, height data, gravimetric database* (in Polish), Report No 11, Space Research Center, Polish Academy of Sciences.

Łyszkowicz A., (2009): *Assessment of accuracy of EGM08 model over the area of Poland*, Technical Reports, Vol.12, pp. 118–134.

Łyszkowicz A., (2010): *Quasigeoid for the area of Poland computed by least squares collocation*, Technical Sciences, No 13, Y 2010.

McCarthy D., Petit G., (eds) (2004): *IERS Conventions 2003*, IERS Technical Note 32, BKG, Frankfurt.

Moritz H., (1980): *Advanced physical geodesy*, Wichmann, Karlsruhe.

Moritz H., (2000): *Geodetic Reference System 1980*, Journal of Geodesy, 74(1), pp. 128–133, doi: 10.1007/s001900050278.

NGA, (1996): *Performance specification Digital Terrain Elevation Data (DTED)*, National Geospatial-Intelligence Agency, Document MIL-PRF-89020A.

Omang O.C.D., Forsberg R., (2000): *How to handle topography in practical geoid determination: three examples*, Journal of Geodesy, 74(6), pp. 458–466.

Pavlis N.K., Holmes S.A., Kenyon S.C., Factor J.K., (2012): *The development and evaluation of the earth gravitational model 2008 (EGM2008)*, J. Geophys. Res., 117(B4), doi: 10.1029/2011JB008916.

Schwarz K.P., Sideris M.G., Forsberg R., (1990): *The use of FFT techniques in physical geodesy*, Geophys. J. Int., 100(3), pp. 485–514, doi: 10.1111/j.1365-246X.1990.tb00701.x.

Torge W., Müller J., (2012): *Geodesy*, 4th edition, Walter de Gruyter, Berlin-Boston. ISBN 978-3-11-020718-7.

Tscherning C., Forsberg R., Knudsen P., (1992): *The GRAVSOFT package for geoid determination*, First Continental Workshop On The Geoid In Europe "Towards a Precise Pan-European Reference Geoid for the Nineties" Prague, 11 – 14 May.

Wichiencharoen C., (1982): *Indirect effects on the computation of the geoid undulation*, OSU Report No 336.

GDQM-PL13 – nowy model quasigeoidy grawimetrycznej dla Polski

Małgorzata Szelachowska
Instytut Geodezji i Kartografii, ul. Modzelewskiego 27, PL 02-679 Warszawa
Tel.: +48 22 3291903, Fax: +48 22 3291950, E-mail: malgorzata.szelachowska@igik.edu.pl

Jan Kryński
Instytut Geodezji i Kartografii, ul. Modzelewskiego 27, PL 02-679 Warszawa
Tel.: +48 22 3291904, Fax: +48 22 3291950, E-mail: jan.krynski@igik.edu.pl

Streszczenie: Opracowano nowy model quasigeoidy grawimetrycznej dla Polski o kryptonimie GDQM-PL13. Jako dane wyjściowe wykorzystano średnie anomalie Faye'a w siatce 1′×1′, odchylenia pionu z obszaru Polski, anomalie grawimetryczne z sąsiednich krajów oraz model EGM2008. Obliczenia wykonano metodą kolokacji najmniejszych kwadratów z wykorzystaniem strategii „remove-compute-restore" (RCR). Anomalie wysokości obliczone z modelu GDQM-PL13 porównano z odpowiadającymi im anomaliami wysokości otrzymanymi z danych satelitarno-niwelacyjnych na punktach sieci POLREF, EUVN i ASG-EUPOS oraz na stacjach kontrolnego trawersu satelitarno-niwelacyjnego. Nowy model quasigeoidy porównano również z modelem quasigeoidy GDQ08 opracowanym dla obszaru Polski w 2008 roku, z modelem EGM2008 oraz z najnowszymi globalnymi modelami geopotencjału opracowanymi z wykorzystaniem danych z misji GOCE. Wyniki porównania poddano gruntownej analizie, a następnie oszacowano dokładność modelu GDQM-PL13.

Słowa kluczowe: model quasigeoidy grawimetrycznej, kolokacja najmniejszych kwadratów, dane satelitarno-niwelacyjne, globalny model geopotencjału